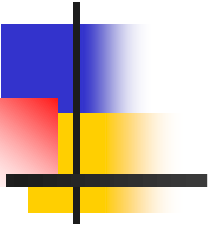


IEng: 5361-Industrial Management and Engineering Economy



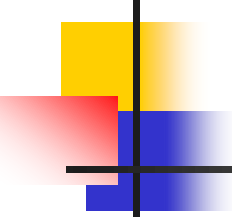
Chapter 3

Forecasting



Learning Objectives

- *Define forecasting and discuss its importance*
- *Discuss different forecasting time horizons*
- *Discuss types, steps & principles of forecasting*
- *Discuss forecasting approaches and techniques*
- *Identify forecasting errors*



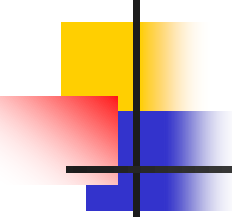
What is forecasting?

- It is a **tool** used for **predicting future value** (demand) based on past information.
- It is a process of **predicting a future event**.
- It is an **underlying basis** of **all business decisions** such as Production, Inventory, Personnel, Marketing, Facilities etc.



What forecasting realities ?

- Forecasts are seldom perfect.
- Most techniques assume an underlying stability in the system.
- Product family and aggregated forecasts are more accurate than individual product forecasts.



What forecasting importance ?

■ Marketing managers:

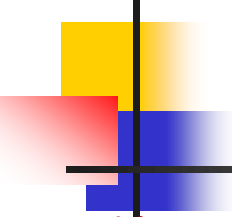
- Use sales forecasts to determine optimal sales force allocations.
- Set sales goals.
- Plan promotions and advertising.

■ Planning for capital investments:

- Predictions about future economic activity.
- Estimating cash inflows accruing from the investment.

■ The personnel department:

- Planning for human resources.



What forecasting importance ?

- *Managers of nonprofit institutions:*

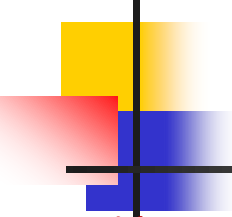
- Forecasts for budgeting purposes.

- *Universities:*

- Forecast student enrollments.
- Cost of operations.
- Funds to be provided by tuition and by government appropriations.

- *The bank has to forecast:*

- Demands of various loans and deposits
- Money and credit conditions so that it can determine the cost of money it lends.



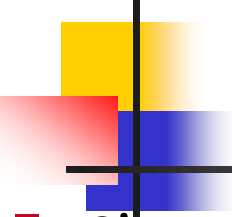
What forecasting importance ?

■ *Manufacturers:*

- Worker absenteeism
- Machine availability
- Material costs
- Transportation and production lead times, etc.

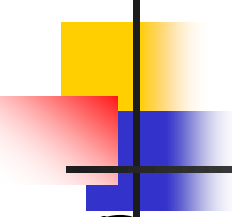
■ *Service providers:*

- Forecasts of population
- Demographic variables
- Weather, etc.



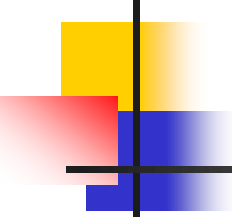
What Forecasting Time Horizons?

- Short-range forecast
 - Up to 1 year, usually less than 3 months
 - **Daily demand and resource requirements, job scheduling, workforce levels, job assignments, production levels**
- Medium-range forecast
 - 3 months to 3 years
 - **Sales and production planning**, budgeting, secure additional resources for upcoming year, reflect peaks & valley in demand
- Long-range forecast
 - 3+ years
 - New product planning, build new **facilities, secure long-term financing**, research and development



What types of forecasting?

- Economic forecasts
 - Address business cycle – inflation rate, money supply, housing starts, etc.
- Technological forecasts
 - Predict rate of technological progress
 - Impacts development of new products
- Demand forecasts
 - Predict sales of existing product



What steps in forecasting?

There are **seven basic steps** in the forecasting process:

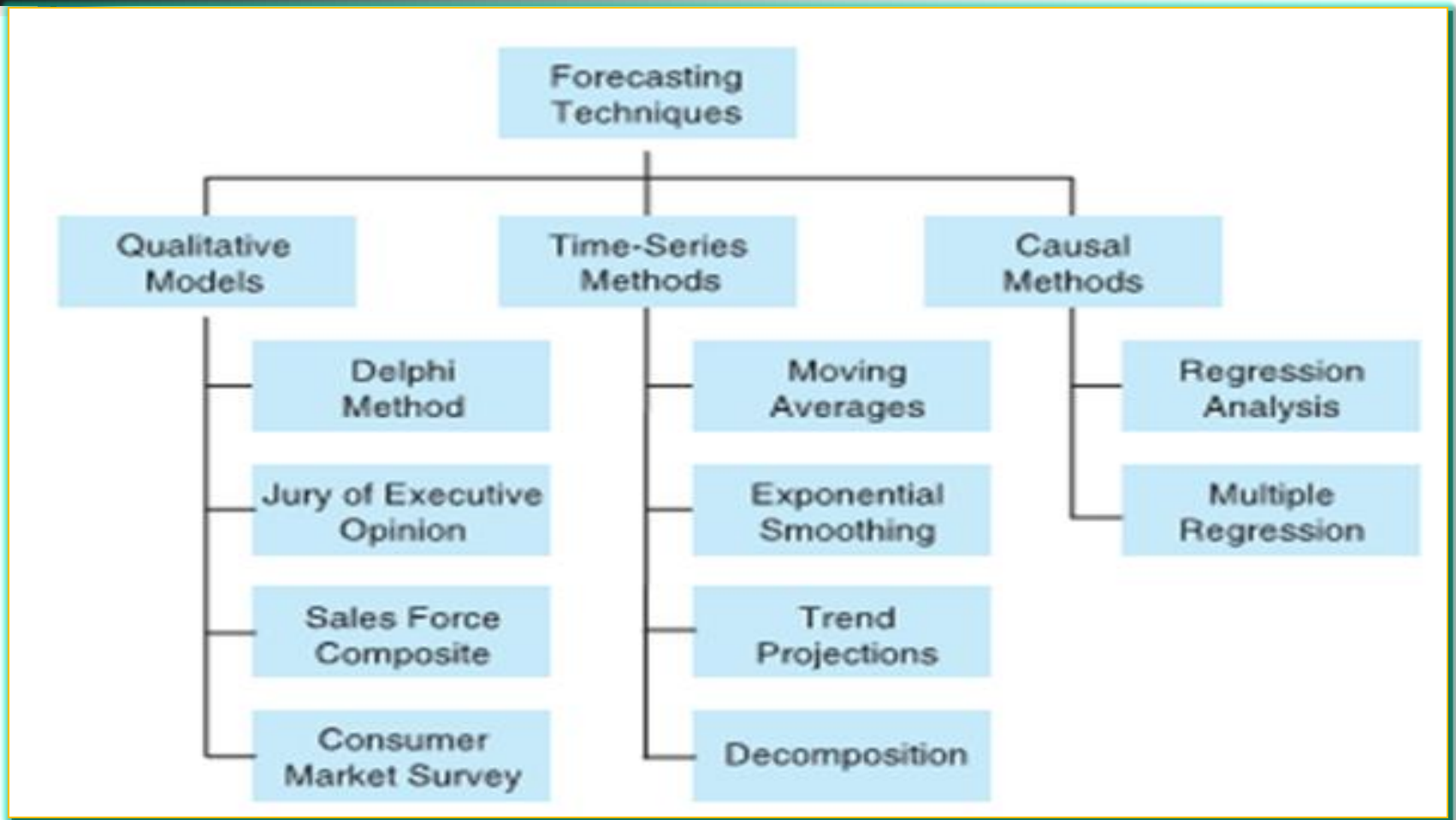
1. Determine the purpose of the forecast.
2. Select the items to be forecast.
3. Establish a time horizon.
4. Select the forecasting technique.
5. Gather and analyze relevant data.
6. Prepare the forecast.
7. Monitor the results.

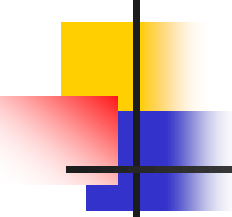


What forecasting principles?

- Different types of forecasting models are differ in their degree of complexity, the amount of data they use, and the way they generate the forecast.
- However, the following are **common features to all** forecasting models:
 - Forecasts are rarely perfect: it is almost impossible to make a perfect prediction.
 - Forecasts are more accurate for groups or families of items rather than for individual items.
 - Forecasts are more accurate for shorter than longer time horizons.

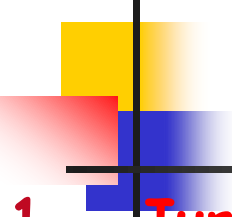
What forecasting approaches?





What forecasting approaches?

- **Qualitative methods – judgmental methods**
 - Forecasts generated subjectively by the forecaster
 - Educated guesses
 - Used when little data exist
 - New products
 - New technology
 - Involves intuition, experience
 - e.g., forecasting sales on Internet



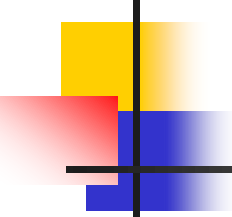
What forecasting approaches?

1. Jury of executive opinion:

- Pool opinions of high-level experts, sometimes augment by statistical models.
- The subjective views of executives or experts from sales, production, finance, purchasing, and administration are averaged to generate a forecast about future sales.

2. Delphi method:

- Panel of experts, queried iteratively until consensus is reached
- requires one person to administer and coordinate the process and poll the team members (respondents) through a series of sequential questionnaires.



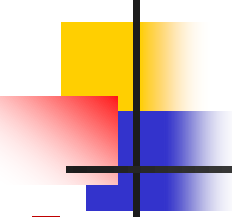
What forecasting approaches?

2. Sales force composite:

- Estimates from individual salespersons are reviewed for reasonableness, then aggregated.

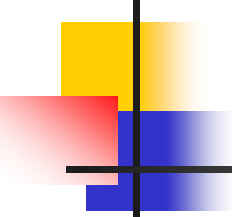
3. Consumer Market Survey

- Ask the customer.
- Some companies conduct their own market surveys regarding specific consumer purchases.
- Surveys may consist of telephone contacts, personal interviews, or questionnaires as a means of obtaining data.



What forecasting approaches?

- **Quantitative methods – based on mathematical modeling**
 - Forecasts generated through mathematical modeling
 - Used when situation is 'stable' and historical data exist
 - Existing products
 - Current technology
 - Involves mathematical techniques
 - e.g., forecasting sales of LCD televisions



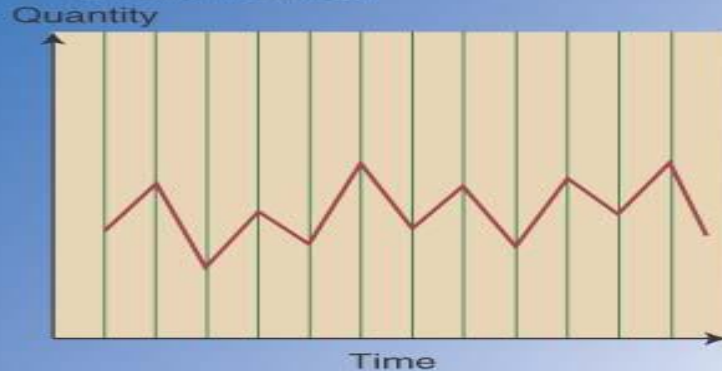
What forecasting approaches?

- **Historic pattern** or data may exhibit one of the following:
 - **Level (long-term average)** - data fluctuates around a constant mean.
 - **Trend** - data exhibits an increasing or decreasing pattern
 - **Seasonality**- effects are similar variations occurring during corresponding periods, can be quarterly, monthly, weekly, daily, or even hourly indexes.
 - **Cycle** - are the long-term swings about the trend line.
 - **Irregular variations** - caused by unusual circumstances
 - **Random variations** - caused by chance, cannot be predicted

What forecasting approaches?

■ Time Series Patterns

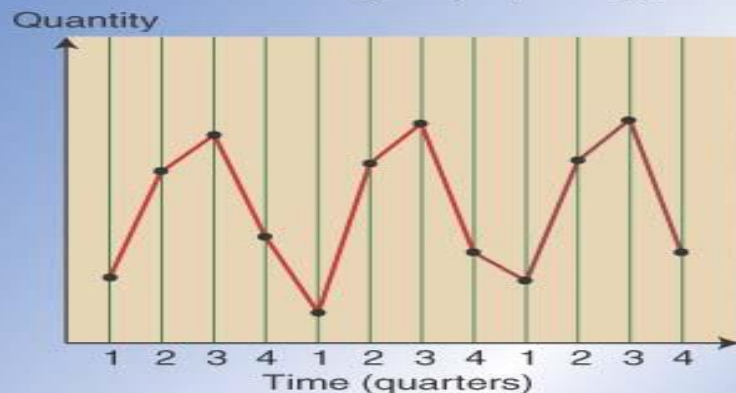
(a) Level or Horizontal Pattern:
Data follows a horizontal pattern around the mean



(b) Trend Pattern:
Data is progressively increasing (shown) or decreasing



(c) Seasonal Pattern:
Data exhibits a regularly repeating pattern

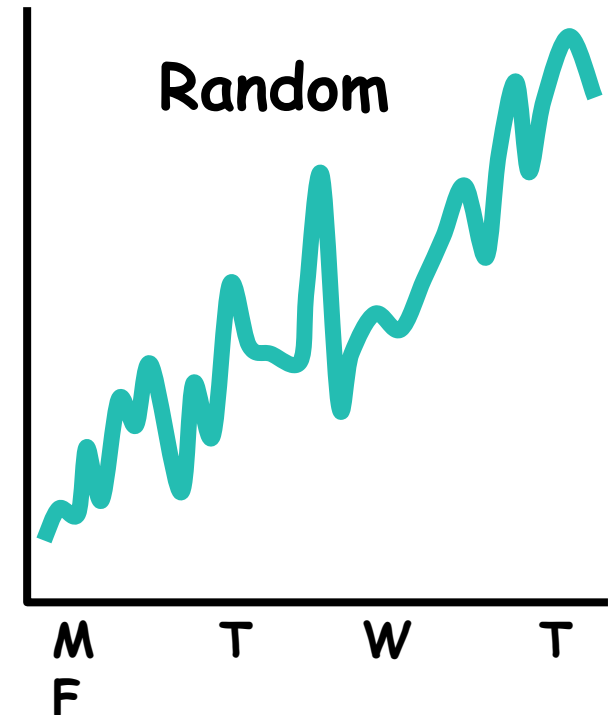
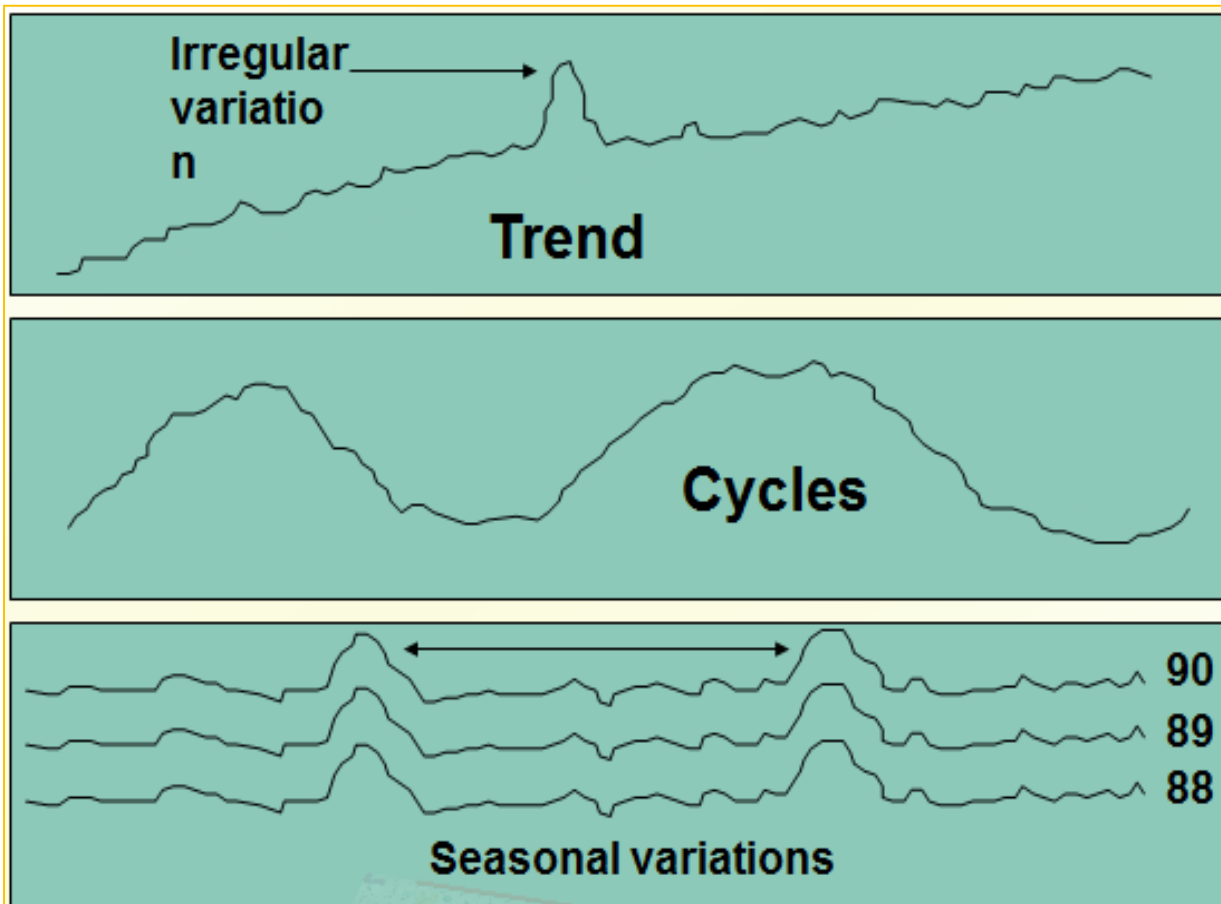


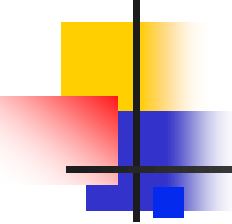
(d) Cycle:
Data increases or decreases over time



What forecasting approaches?

Time Series Patterns





What forecasting approaches?

Time Series Models

1. Naive (projection) approach
2. Simple average
3. Moving average
4. Weighted moving average
5. Exponential smoothing

Time - series
models

6. Linear regression (trend projection)
7. Multiple regression

Associative
(causal) models



What forecasting approaches?

Naive (projection):

- Assumes demand in next period is the same as demand in most recent period
- The forecast for the period t , F_t , is simply a projection of previous period $t-1$ demand, A_{t-1} .
 - $F_t = A_{t-1}$
 - **Example:** If the actual demand of period t is **120**, then the forecast of the period $t+1$ is **120**.
- Although this method is easy to use, **doesn't make use of data** that is easily available to most managers; thus, using more of the **historical data should improve the forecast**.



What forecasting approaches?

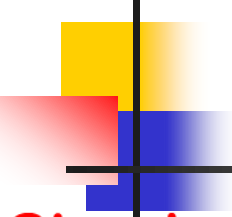
Simple average:

- Demands occurring in all previous time periods is taken as the demand forecast for the next time period in this method.

$$SA = \left(\frac{D1 + D2 + D3}{3} \right)$$

- **Example:** XYZ television supplier found a demand of 200 sets in July, 225 sets in August & 245 sets in September. Find the demand forecast for the month of October using simple average method.
- The average demand for the month of October is

$$\begin{aligned} SA &= \left(\frac{D1 + D2 + D3}{3} \right) \\ &= \left(\frac{200 + 225 + 245}{3} \right) \\ &= 223.33 \\ &\approx 224 \text{ units} \end{aligned}$$



What forecasting approaches?

Simple Moving Average (MA):

- The simple moving average forecast makes use of more of the historical demand data.
- MA is a series of arithmetic means & **Used if little or no trend**
- On shortcoming of MA is the equal weighting of data.
- An n-period moving average uses the last n periods of demand as a forecast for next periods demand:

$$F_t = \frac{A_{t-1} + A_{t-2} + \cdots + A_{t-n}}{n}$$

- Where n = total number of periods in the average,

F_t = Forecast for period t ,

$A_{t-1}, A_{t-2}, \dots, A_{t-n}$ = Actual occurrences for previous periods 1, 2, ..., n .

What forecasting approaches?

■ Example:

W e e k	D e m a n d
1	6 5 0
2	6 7 8
3	7 2 0
4	7 8 5
5	8 5 9
6	9 2 0
7	8 5 0
8	7 5 8
9	8 9 2
1 0	9 2 0
1 1	7 8 9
1 2	8 4 4

- *What are the 3-week and 6-week moving average forecasts for demand?*
- *Which forecast would you prefer?*

$$F_t = \frac{A_{t-1} + A_{t-2} + \cdots + A_{t-n}}{n}$$

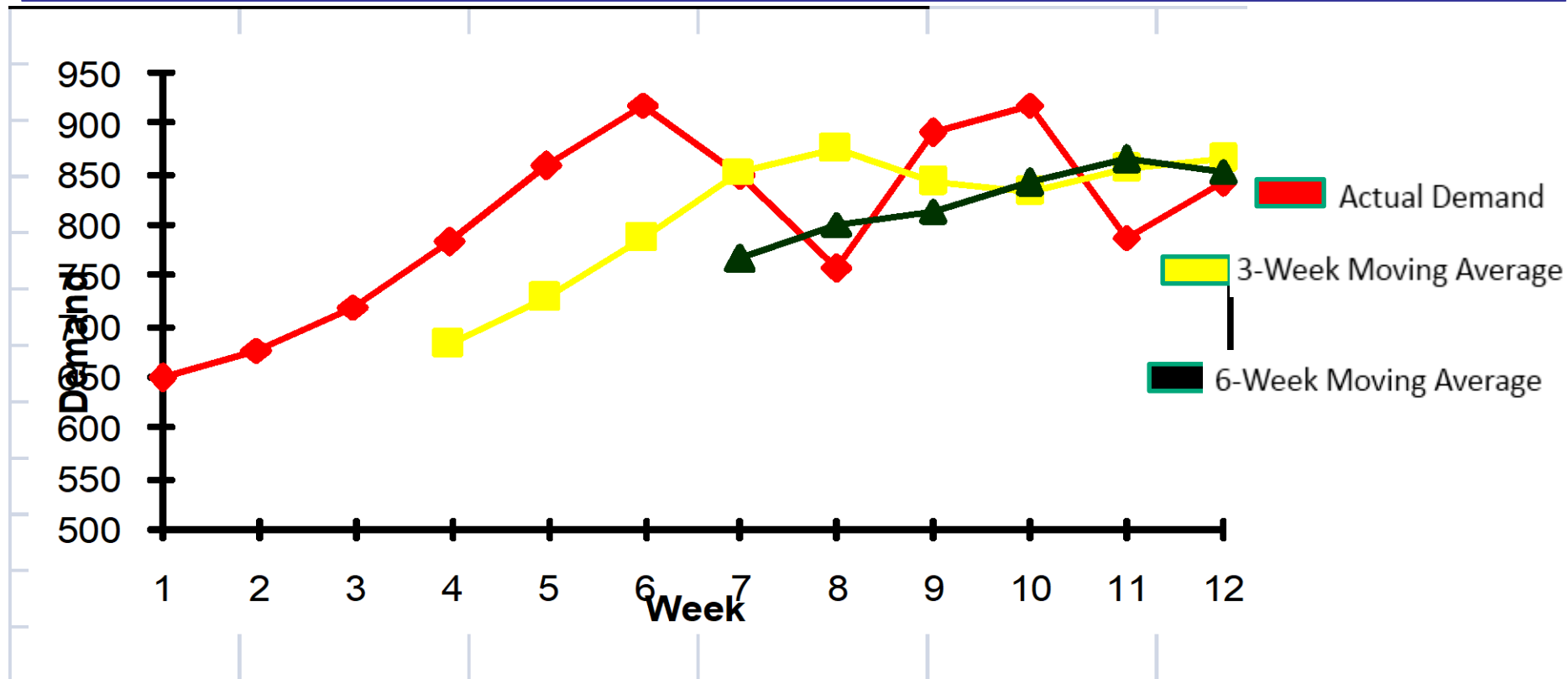
What forecasting approaches?

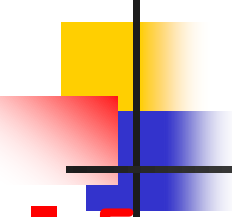
- Calculating the moving averages gives us:

Week	Demand	3-Week	6-Week	
1	650	$F_4 = (650 + 678 + 720) / 3$ $= 682.67$		
2	678			
3	720			
4	785	682.67	$F_7 = (650 + 678 + 720 + 785 + 859 + 920) / 6$ $= 768.67$	
5	859	727.67		
6	920	788.00		
7	850	854.67	768.67	
8	758	876.33	802.00	
9	892	842.67	815.33	
10	920	833.33	844.00	
11	789	856.67	866.50	
12	844	867.00	854.83	

What forecasting approaches?

- Plotting the moving averages and comparing them shows how the lines smooth out to reveal the overall upward trend in this example

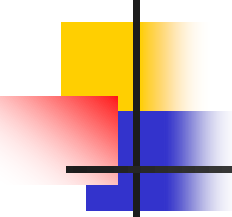




What forecasting approaches?

- **Exercise:** A company sells storage shed, Determine the forecast of January using 3 month simple moving average.)

MONTH	ACTUAL SHED SALES
January	10
February	12
March	13
April	16
May	19
June	23
July	26
August	30
September	28
October	18
November	16
December	14
January	—



What forecasting approaches?

Potential Problems With Moving Average

- Increasing n smooth the forecast but makes it less sensitive to changes.
- **Do not forecast trends well.**
- Require extensive historical data.



What forecasting approaches?

Weighted Moving Average:

- A weighted moving average is a moving average where each historical demand may be weighted differently.
- Used when some trend might be present & Older data usually less important.
- The most recent data is the most relevant.
- Thus, the weighted moving average allows for more emphasis to be placed on the most recent data. This forecast is:

$$F_t = \frac{w_{t-1}A_{t-1} + w_{t-2}A_{t-2} + \cdots + w_{t-n}A_{t-n}}{w_{t-1} + w_{t-2} + \cdots + w_{t-n}}$$



What forecasting approaches?

- Where w_{t-1} is the weight applied to the actual demand incurred during period $t-1$, and so on.
- Intuitively, the expectation would be that the **more recent demand data should be weighted more heavily** than older data; so, generally, one would expect the weights to follow the relationship $w_t \geq w_{t-1} \geq w_{t-2} \geq \dots$
- The sum of the weights is **one**.

What forecasting approaches?

- **Example:** depending on the following weekly demand and weights, what is the forecast for the 4th period or Week 4?

Week	Demand
1	650
2	678
3	720
4	

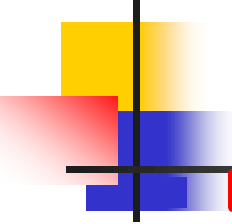
Weights:

0.5 0.3 0.2

■ Solution

$$F_4 = 0.5(720) + 0.3(678) + 0.2(650) = 693.4$$

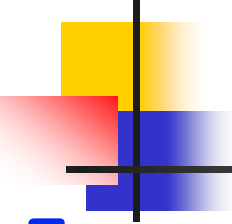
week	Demand	Forecast
1	650	
2	678	
3	720	
4		693.4



What forecasting approaches?

Exercise: Consider the weights $3/6$, $2/6$, $1/6$ for periods t-1, t-2 and t-3 respectively. Determine the forecast of January & compare its smoothness with moving average of the same data on common plot.

MONTH	ACTUAL SHED SALES
January	10
February	12
March	13
April	16
May	19
June	23
July	26
August	30
September	28
October	18
November	16
December	14
January	—



What forecasting approaches?

Exponential smoothing:

- It is a Form of weighted moving average, **Weights decline exponentially** & Most recent data weighted most.
- Nice properties of a weighted moving average would be one where the **weights not only decrease as older** and older data are used, but one where the **differences between the weights** are “**smooth**”.
- Involves **little record keeping** of past data.
- Smoothing constant (α) Ranges from **0 to 1** & **Subjectively chosen**.



What forecasting approaches?

New forecast = α (Last period's forecast)
+ (Last period's actual demand
– Last period's forecast)

$$F_t = F_{t-1} + \alpha (A_{t-1} - F_{t-1})$$

Where: F_t = new forecast

A_{t-1} = Actual demand

F_{t-1} = previous forecast

α = smoothing (or weighting)
constant ($0 \leq \alpha \leq 1$)

What forecasting approaches?

Example 1:

Week	Demand
1	820
2	775
3	680
4	655
5	

- What are the exponential smoothing forecasts for periods 2 to 5 using $\alpha = 0.5$? Assume $F_1 = D_1$

Solution

$$F_3 = 820 + (0.5)(775 - 820) = 797.5$$

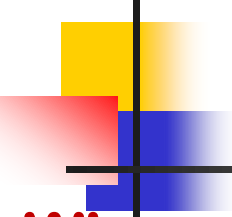
week	Demand	Forecast ($\alpha = 0.5$)
1	820	820
2	775	820
3	680	797.5
4	655	738.75
5	?	696.88

What forecasting approaches?

Example 2:

- The prediction of the future depends mostly on the **most recent observation**, and on the **error** for the latest forecast.

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST USING $\alpha = 0.10$	FORECAST USING $\alpha = 0.50$
1	180	175	175
2	168	$175.5 = 175.00 + 0.10(180 - 175)$	177.5
3	159	$174.75 = 175.50 + 0.10(168 - 175.50)$	172.75
4	175	$173.18 = 174.75 + 0.10(159 - 174.75)$	165.88
5	190	$173.36 = 173.18 + 0.10(175 - 173.18)$	170.44
6	205	$175.02 = 173.36 + 0.10(190 - 173.36)$	180.22
7	180	$178.02 = 175.02 + 0.10(205 - 175.02)$	192.61
8	182	$178.22 = 178.02 + 0.10(180 - 178.02)$	186.30
9	?	$178.60 = 178.22 + 0.10(182 - 178.22)$	184.15



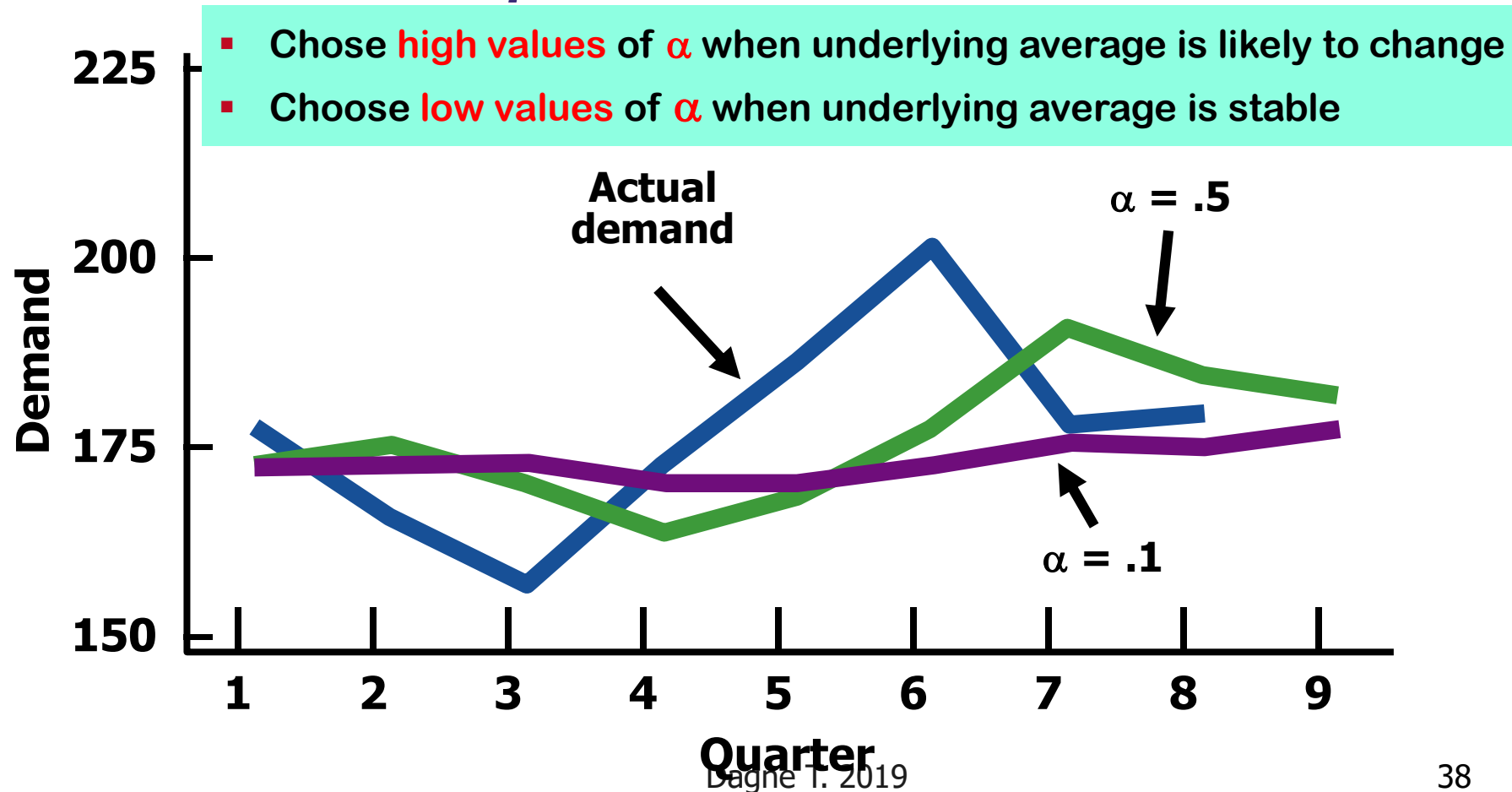
What forecasting approaches?

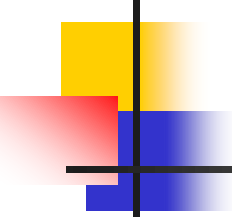
Why use exponential smoothing?

- Uses less storage space for data
 - More accurate
 - Easy to understand
 - Little calculation complexity
- The smoothing constant α expresses how much our forecast will react to observed differences.
- If α is low: there is little reaction to differences.
 - If α is high: there is a lot of reaction to differences.

What forecasting approaches?

Impact of Different α





What forecasting approaches?

Selecting Smoothing Constant (α):

- The appropriate value of the smoothing constant, α , however, **can make the difference** between an **accurate forecast** and an **inaccurate forecast**.
- In picking a value for the smoothing constant, the objective is to **obtain the most accurate forecast**.
- Several values of the smoothing constant may be **tried**, and the one with **the lowest MAD** (Mean Absolute Deviation) could be selected.

What forecasting approaches?

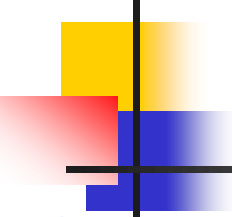
- Selecting smoothing constant with **lowest MAD**.

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = 0.10$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.10$	FORECAST WITH $\alpha = 0.50$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.50$
1	180	175	5	175	5
2	168	175.5	7.5	177.5	9.5
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	<u>3.78</u>	186.30	<u>4.3</u>
Sum of absolute deviations			82.45		98.63

$$MAD = \frac{\sum_{t=1}^n |A_t - F_t|}{n}$$

$$MAD = \frac{\sum |deviation|}{n} = 10.31$$

$$MAD = 12.33$$



What forecasting approaches?

Selecting the Right Forecasting Model

■ Selecting the right forecasting methods depends on:

1. The amount & type of available data

- Some methods require more data than others

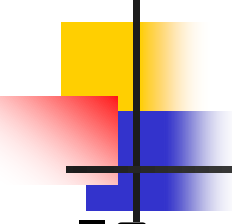
2. Degree of accuracy required

- Increasing accuracy means more data

3. Length of forecast horizon

- Different models for 3 month vs. 10 years

4. Presence of data patterns



What Measures of Forecasting Errors?

- Forecasts are never perfect
- Need to know how much we should **rely on our chosen forecasting method.**
- Measuring **forecast error**: $E_t = A_t - F_t$
- Note that:
 - **Over-forecasts** = negative errors
 - **Under-forecasts** = positive errors.
- Large values of negative or positive errors shows there is **bias in the forecast.**

What Measures of Forecasting Errors?

- Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{t=1}^n |A_t - F_t|}{n}$$

- Cumulative Forecast Error (CFE)

- Also called running sum of forecast error (RSFE)
- Measures any bias in the forecast

$$CFE = \sum (\text{actual} - \text{forecast}) \text{ or}$$

$$RSFE = \sum_{i=1}^n (A_t - F_t)$$

- Mean Square Error (MSE)

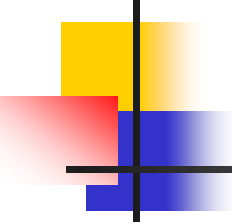
- Penalizes larger errors

$$MSE = \frac{\sum_{t=1}^n (A_t - F_t)^2}{n}$$

- Mean Absolute Percent Error (MAPE)

MAPE =

$$\frac{\sum_{i=1}^n 100 | \text{Actual}_i - \text{Forecast}_i | / \text{Actual}_i}{n}$$



What Measures & comparison of Forecasting Errors?

Example:

- During the past 8 quarters, the Port of Baltimore has unloaded large quantities of grain from ships. The Port's **Operations Manager** wants to test the forecasting method *exponential smoothing* to see how well this method works in predicting tonnage unloaded. He guesses that the forecast of grain unloaded in the first quarter was 175 tons.

Dagne T. 2019

What Measures & comparison of Forecasting Errors?

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
MAD = $\frac{\sum \text{deviations} }{n}$ For $\alpha = .10$ $= 82.45/8 = 10.31$ For $\alpha = .50$ $= 98.62/8 = 12.33$			5.00	175	5.00
			7.50	177.50	9.50
			5.75	172.75	13.75
			1.82	165.88	9.12
			6.64	170.44	19.56
			29.98	180.22	24.78
			1.98	192.61	12.61
			3.78	186.30	4.30
			82.45		98.62

What Measures & comparison of Forecasting Errors?

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
MSE = $\frac{\sum (\text{forecast errors})^2}{n}$ For $\alpha = .10$ = 1,526.54/8 = 190.82 For $\alpha = .50$ = 1,561.91/8 = 195.24			5.00	175	5.00
			7.50	177.50	9.50
			5.75	172.75	13.75
			1.82	165.88	9.12
			6.64	170.44	19.56
			29.98	180.22	24.78
			1.98	192.61	12.61
			3.78	186.30	4.30
			82.45		98.62
		MAD	10.31		12.33

What Measures & comparison of Forecasting Errors?

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
			5.00	175	5.00
			7.50	177.50	9.50
			15.75	172.75	13.75
			1.82	165.88	9.12
			16.64	170.44	19.56
			29.98	180.22	24.78
			1.98	192.61	12.61
			3.78	186.30	4.30
			82.45		98.62
		MAD	10.31		12.33
		MSE	190.82		195.24

$$MAPE = \frac{\sum_{i=1}^n 100 |deviation_i| / actual_i}{n}$$

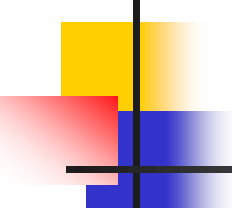
For $\alpha = .10$

$= 44.75/8 = 5.59\%$

For $\alpha = .50$

$= 54.05/8 = 6.76\%$

10.31
190.82
5.59%



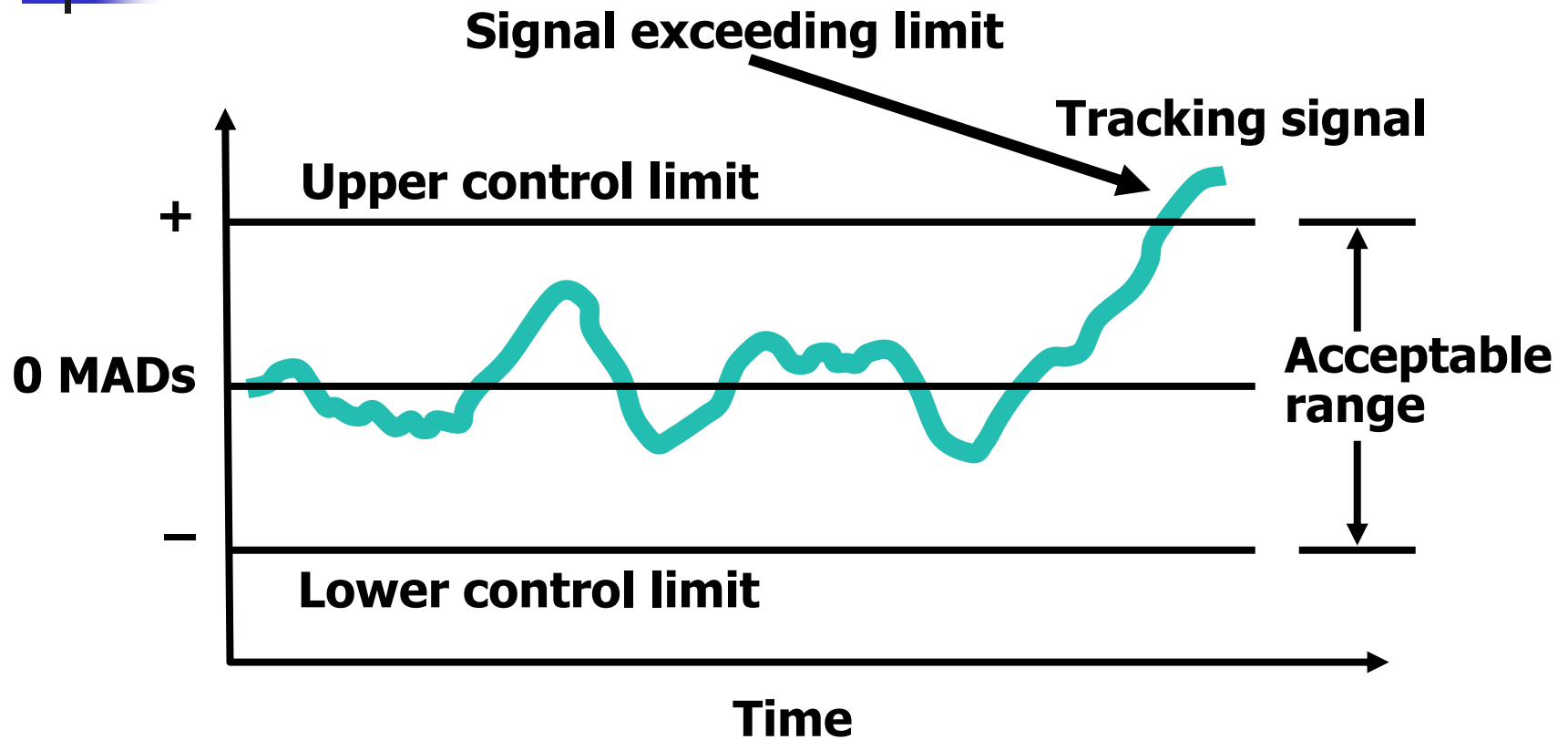
What Measuring Accuracy?

- The **tracking signal** is a measure of how often our estimations have been above or below the actual value.
- It is used to decide when to re-evaluate using a model.

$$\text{RSFE} = \sum_{i=1}^n (A_t - F_t) \qquad \text{TS} = \frac{\text{RSFE}}{\text{MAD}}$$

- **Positive tracking signal**: most of the time actual values are above our forecasted values.
- **Negative tracking signal**: most of the time actual values are below our forecasted values.

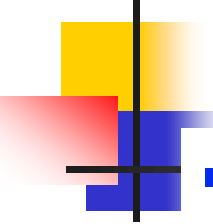
What Measuring Accuracy?



What Measuring Accuracy?

Example 1:

S.N	Actual	Naïve	Simple (n=3)	Weighted (n=3, t-1=0.45, t-2=0.35, t-3=0.2)	Exponential ($\alpha=0.1$)	Exponential ($\alpha=0.5$)	Exponential ($\alpha=0.8$)
1	110				105	105	105
2	100	110.0			105.5	107.5	109.0
3	120	100.0			105.0	103.8	101.8
4	140	120.0	110.0	108.5	106.5	111.9	116.4
5	170	140.0	120.0	115.0	109.8	125.9	135.3
6	150	170.0	143.3	137.0	115.8	148.0	163.1
7	160	150.0	153.3	152.5	119.2	149.0	152.6
8	190	160.0	160.0	161.0	123.3	154.5	158.5
9	200	190.0	166.7	161.5	130.0	172.2	183.7
10	190	200.0	183.3	178.5	137.0	186.1	196.7
11		190.0	193.3	193.5	142.3	188.1	191.3
	MAD	17.8	23.3	26.6	38.4	18.1	16.6
	CFE	80.0	163.3	186.0	372.9	166.1	107.9
	TS	4.50	7.00	7.00	9.71	9.17	6.52



What Measuring Accuracy?

■ Example 2:

Qtr	Actual Demand	Forecast Demand	Error	Cumm Error	Absolute Forecast Error	Cumulative Absolute Forecast Error	MAD
1	90	100	-10	-10	10	10	10.0
2	95	100	-5	-15	5	15	7.5
3	115	100	+15	0	15	30	10.0
4	100	110	-10	-10	10	40	10.0
5	125	110	+15	+5	15	55	11.0
6	140	110	+30	+35	30	85	14.2

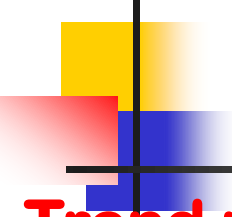
What Measuring Accuracy?

Example 2:

Qtr	Actual Demand	Forecast Demand	Error	Cumm Error	Absolute Forecast Error	Cumulative Absolute Forecast Error	MAD
1			-10	-10	10	10	10.0
2			-5	-15	5	15	7.5
3			+15	0	15	30	10.0
4			-10	-10	10	40	10.0
5			+15	+5	15	55	11.0
6			+30	+35	30	85	14.2

Tracking Signal (Cumm Error/MAD)							
$-10/10 = -1$							
$-15/7.5 = -2$							
$0/10 = 0$							
$-10/10 = -1$							
$+5/11 = +0.5$							
$+35/14.2 = +2.5$							

The variation of the tracking signal between -2.0 and +2.5 is within acceptable limits



What forecasting approaches?

Trend projection (linear regression)

- Causal models establish a **cause-and-effect relationship** between independent and dependent variables.
- Fitting a trend line to historical data points to project into the medium to long-range
- Linear trends can be found using the **least squares** technique

$$\hat{y} = a + bx$$

where $\hat{y}$ = computed value of the variable to be predicted
(dependent variable)

a = y -axis intercept

b = slope of the regression line

x = the independent variable

What forecasting approaches?

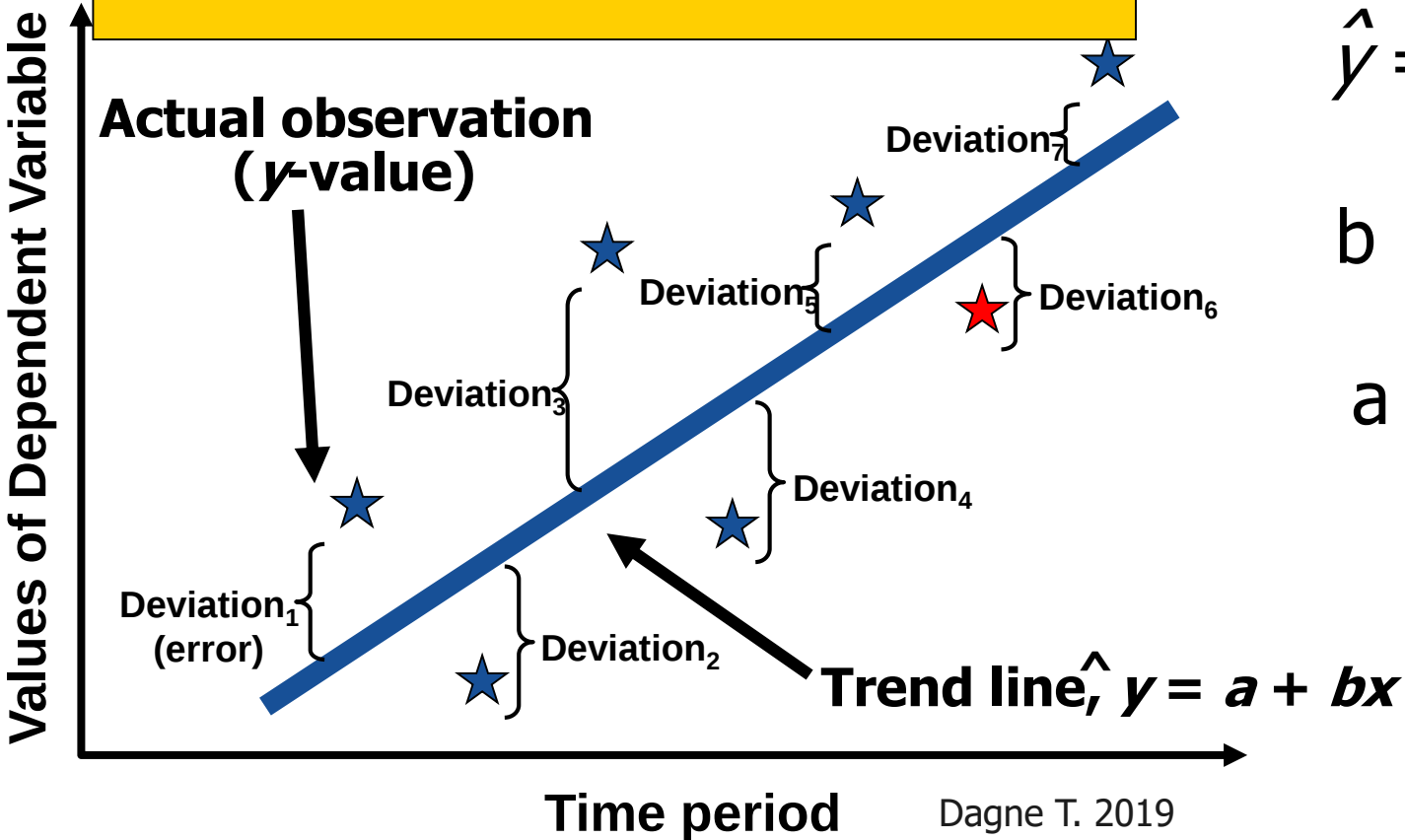
- Least squares method minimizes the sum of the squared errors (deviations)

- Equations to calculate the regression variables

$$\hat{y} = a + bx$$

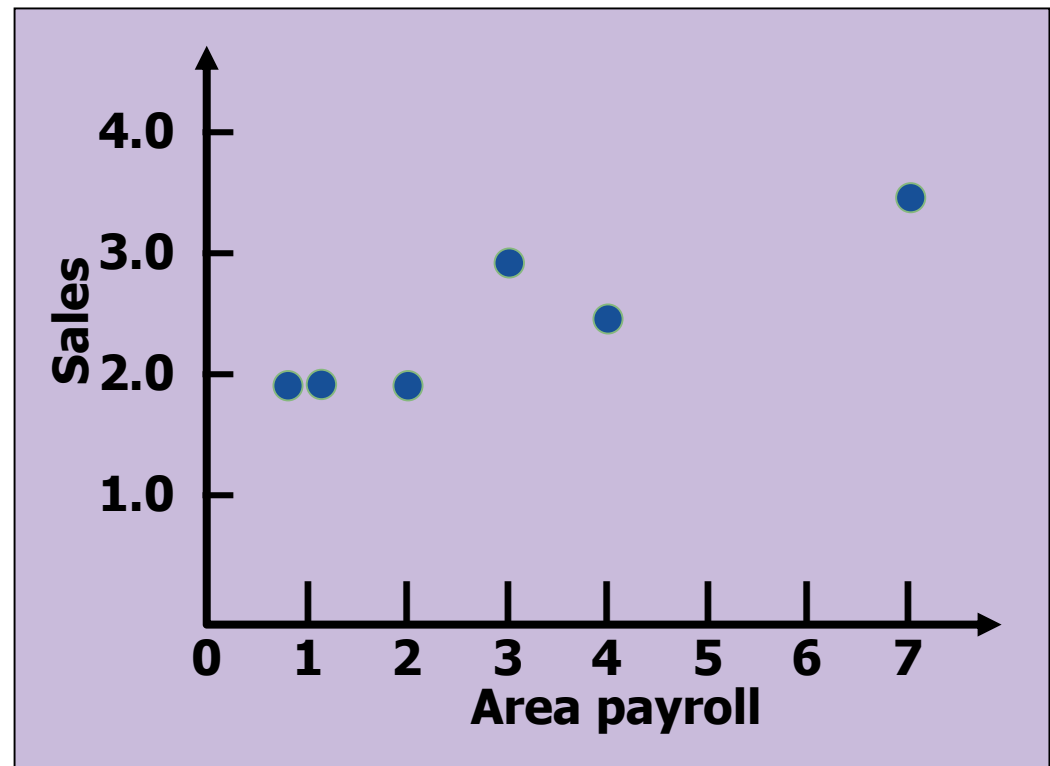
$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$



What forecasting approaches?

Sales (\$ millions), y	Area Payroll (\$ billions), x
2.0	1
3.0	3
2.5	4
2.0	2
2.0	1
3.5	7



What forecasting approaches?

Sales, y	Payroll, x	x^2	xy
2.0	1	1	2.0
3.0	3	9	9.0
2.5	4	16	10.0
2.0	2	4	4.0
2.0	1	1	2.0
<u>3.5</u>	<u>7</u>	<u>49</u>	<u>24.5</u>
$\Sigma y = 15.0$	$\Sigma x = 18$	$\Sigma x^2 = 80$	$\Sigma xy = 51.5$

$$\bar{x} = \Sigma x / 6 = 18 / 6 = 3 \quad b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \frac{51.5 - (6)(3)(2.5)}{80 - (6)(3^2)} = .25$$

$$\bar{y} = \Sigma y / 6 = 15 / 6 = 2.5 \quad a = \bar{y} - b\bar{x} = 2.5 - (.25)(3) = 1.75$$

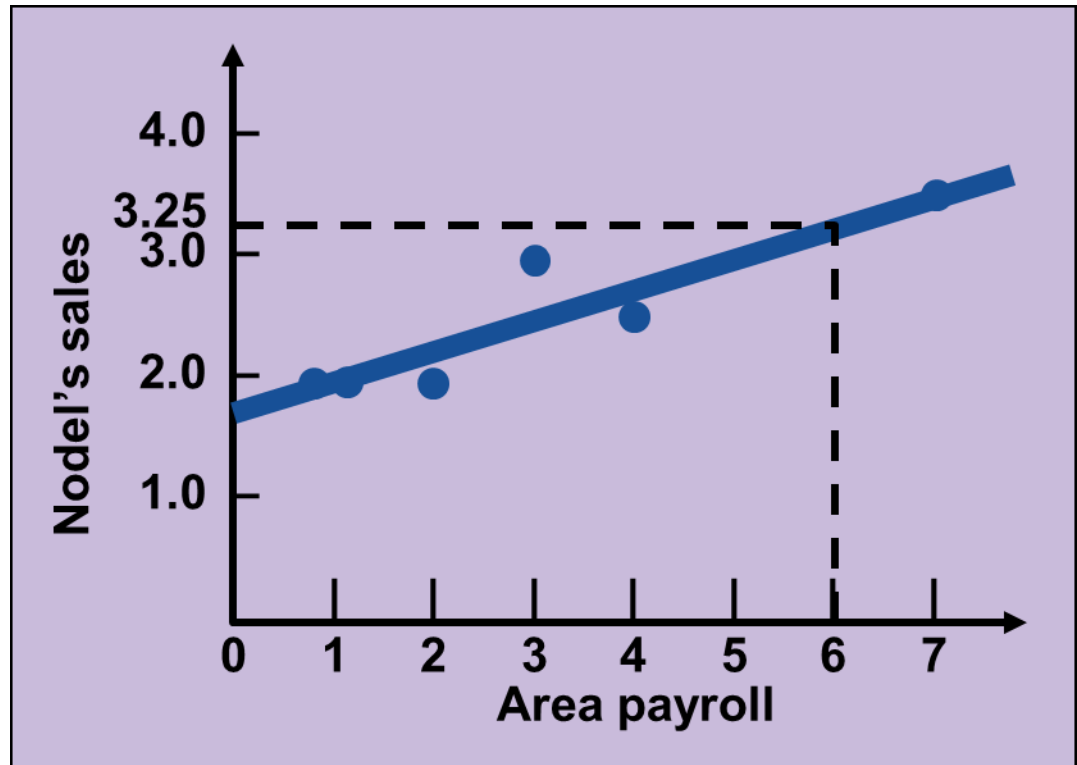
What forecasting approaches?

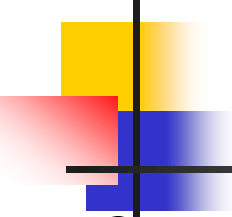
$$\hat{y} = 1.75 + .25x$$

$$\text{Sales} = 1.75 + .25(\text{payroll})$$

If payroll next year is estimated to be \$6 billion, then:

$$\begin{aligned}\text{Sales} &= 1.75 + .25(6) \\ \text{Sales} &= \$3,250,000\end{aligned}$$





What forecasting approaches?

- Correlation coefficient (**r**) measures the **direction and strength of the linear relationship** between two variables.
- The **closer the r value is to 1.0** the better the regression line fits the data points.

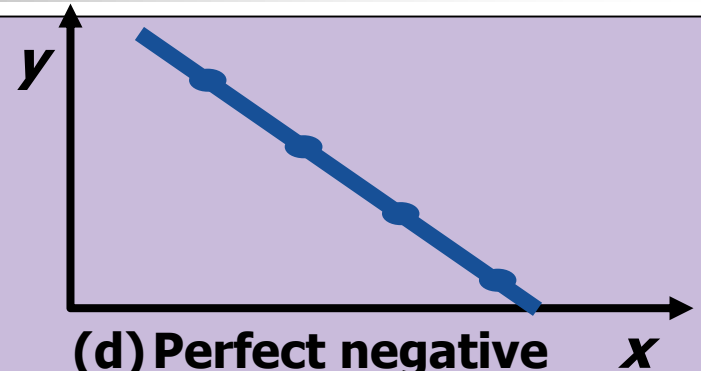
$$r = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{(\sum X^2) - (\sum X)^2} * \sqrt{n(\sum Y^2) - (\sum Y)^2}}$$
$$r = \frac{6(51.5) - 18(15)}{\sqrt{6(80) - (324)} * \sqrt{6(39.5) - (225)}} = 0.901$$
$$r^2 = (0.901)^2 = 0.812$$

- **Coefficient of determination (**r²**)** measures the amount of **variation** in the dependent variable about its **mean** that is explained by the regression line.
- Values of (**r²**) close to 1.0 are desirable.

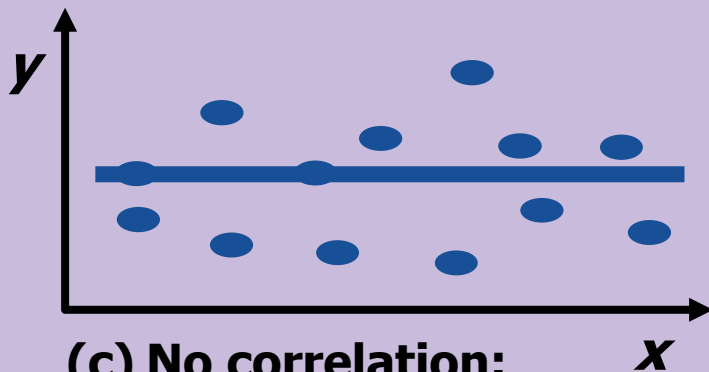
What forecasting approaches?



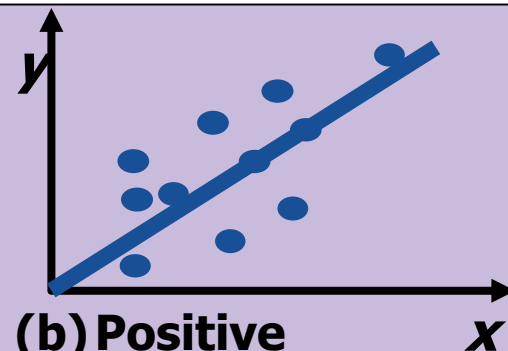
(a) Perfect positive correlation:
 $r = +1$



(d) Perfect negative correlation:
 $r = -1$



(c) No correlation:
 $r = 0$



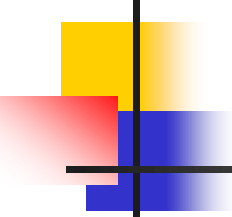
(b) Positive correlation
 $0 < r < 1$



What forecasting approaches?

Exercise: Determine the trend line equation, correlation and determination coefficients.

Year	Time Period (x)	Electrical Power Demand (megawatt)
2006	1	74
2007	2	79
2008	3	80
2009	4	90
2010	5	105
2011	6	142
2012	7	122



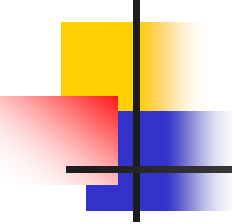
What forecasting approaches?

Multiple Regression Analysis

- It is used when more than one independent variable is to be included in the model.
- It is an **extension of linear regression** to accommodate several independent variables.

$$\hat{y} = a + b_1x_1 + b_2x_2 \dots$$

- Computationally, this is **quite complex** and generally **done on the computer**.



What forecasting approaches?

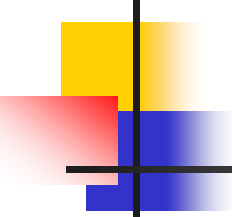
- In the Nodel example, including interest rates in the model gives the new equation:

$$\hat{y} = 1.80 + .30x_1 - 5.0x_2$$

- An improved correlation coefficient of $r = .96$ means this model does a better job of predicting the change in construction sales

$$\text{Sales} = 1.80 + .30(6) - 5.0(.12) = 3.00$$

$$\text{Sales} = \$3,000,000$$

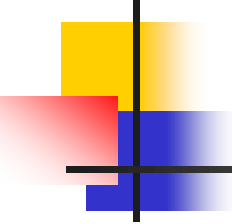


What Seasonal Variations In Data?

- The multiplicative seasonal model can adjust trend data for seasonal variations in demand (jet skis, snow mobiles).

Steps in the process:

1. Find **average** historical demand **for each season**
2. Compute the **average** demand **over all seasons**
3. Compute a **seasonal index** for each season
4. Estimate next year's total demand
5. Divide this estimate of total demand by the number of seasons, then multiply it by the seasonal index for that season



What Seasonal Variations In Data?

Seasonal Index Example

Month	2010	Demand 2011	2012	Average 2010-2012	Average Monthly	Seasonal Index
Jan	80	85	105	90	94	
Feb	70	85	85	80	94	
Mar	80	93	82	85	94	
Apr	90	95	115	100	94	
May	113	125	131	123	94	
Jun	110	115	120	115	94	
Jul	100	102	113	105	94	
Aug	88	102	110	100	94	
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	72	83	80	94	
Dec	82	78	80	80	94	

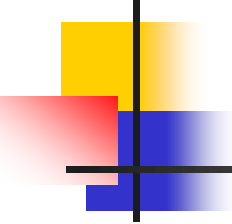
What Seasonal Variations In Data?

Seasonal Index Example

Month	2010	Demand 2011	2012	Average 2010-2012	Average Monthly	Seasonal Index
Jan	80	85	105	90	94	0.957

$$\begin{aligned}\text{Seasonal index} &= \frac{\text{Average 2010-2012 monthly demand}}{\text{Average monthly demand}} \\ &= 90/94 = .957\end{aligned}$$

Sept	85	90	95	90	94
Oct	77	78	85	80	94
Nov	75	72	83	80	94
Dec	82	78	80	80	94



What Seasonal Variations In Data?

Seasonal Index Example

Month	2010	Demand 2011	2012	Average 2010-2012	Average Monthly	Seasonal Index
Jan	80	85	105	90	94	0.957
Feb	70	85	85	80	94	0.851
Mar	80	93	82	85	94	0.904
Apr	90	95	115	100	94	1.064
May	113	125	131	123	94	1.309
Jun	110	115	120	115	94	1.223
Jul	100	102	113	105	94	1.117
Aug	88	102	110	100	94	1.064
Sept	85	90	95	90	94	0.957
Oct	77	78	85	80	94	0.851
Nov	75	72	83	80	94	0.851
Dec	82	78	80	80	94	0.851

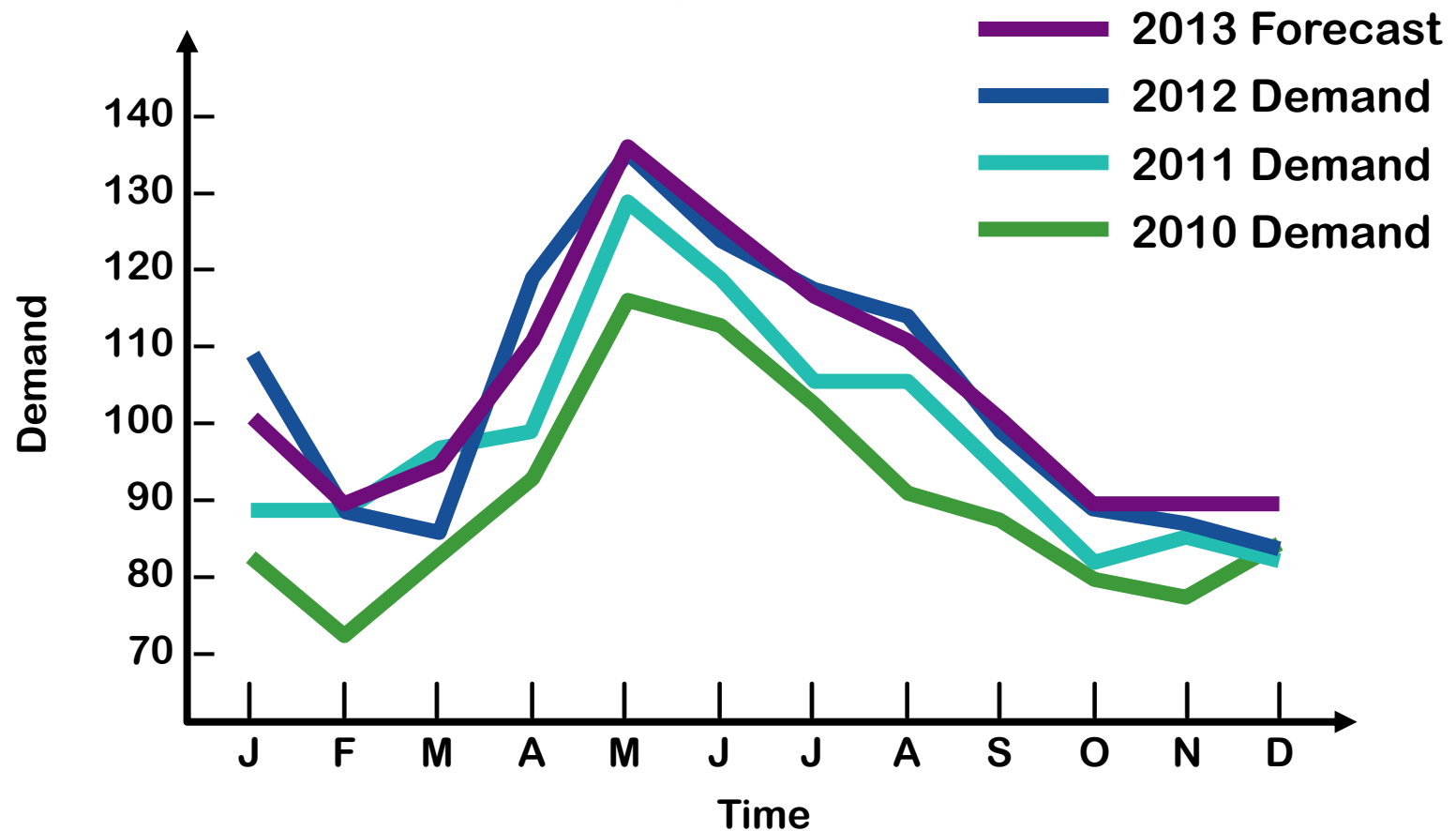
What Seasonal Variations In Data?

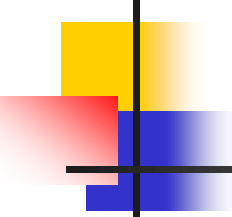
Seasonal Index Example

Month	2010	Demand	2012	Average	Average	Seasonal				
Monthly	Index	2011		2010-2012						
Jan	80	85	105	90	94	0.957				
Feb	70	Forecast for 2013 Expected annual demand = 1,200 Jan $\frac{1,200}{12} \times .957 = 96$ Feb $\frac{1,200}{12} \times .851 = 85$				94	0.851			
Mar	80					94	0.904			
Apr	90					94	1.064			
May	113					94	1.309			
Jun	110					94	1.223			
Jul	100					94	1.117			
Aug	88					94	1.064			
Sept	85					94	0.957			
Oct	77					94	0.851			
Nov	75					94	0.851			
Dec	82					78	80	80	94	0.851

What Seasonal Variations In Data?

Seasonal Index Example





Exponential Smoothing with Trend Adjustment

- When a trend is present, exponential smoothing must be modified to respond to trend.

Forecast including trend (FIT_t) = Exponentially smoothed forecast (F_t) + Exponentially smoothed trend (T_t)

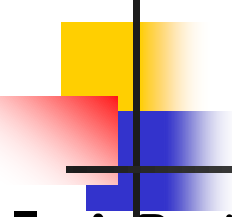
$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1})$$

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$$

Step 1: Compute F_t

Step 2: Compute T_t

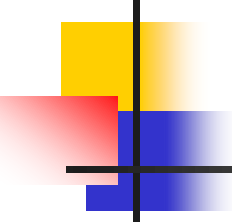
Step 3: Calculate the forecast $FIT_t = F_t + T_t$



Exponential Smoothing with Trend Adjustment

- A Portland manufacturer wants to forecast the demand for a pollution-control equipment. Past data shows that there is an increasing trend. The company assumes the initial forecast for month 1 was 11 units and the trend over that period was 2 units. $\alpha = 0.2$ $\beta = 0.4$.

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17			
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				



Exponential Smoothing with Trend Adjustment

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17			
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Step 1: Forecast for Month 2

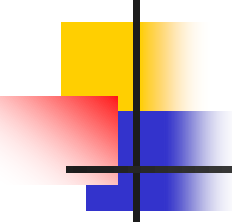
$$\begin{aligned}
 F_2 &= \alpha A_1 + (1 - \alpha)(F_1 + T_1) \\
 F_2 &= (.2)(12) + (1 - .2)(11 + 2) \\
 &= 2.4 + 10.4 = 12.8 \text{ units}
 \end{aligned}$$

Exponential Smoothing with Trend Adjustment

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11		
2	17	12.80	2	13.00
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Step 2: Trend for Month 2

$$\begin{aligned}
 T_2 &= \beta(F_2 - F_1) + (1 - \beta)T_1 \\
 T_2 &= (.4)(12.8 - 11) + (1 - .4)(2) \\
 &= .72 + 1.2 = 1.92 \text{ units}
 \end{aligned}$$

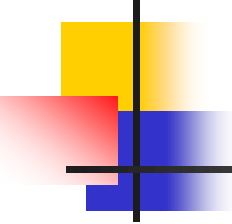


Exponential Smoothing with Trend Adjustment

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Step 3: Calculate FIT for Month 2

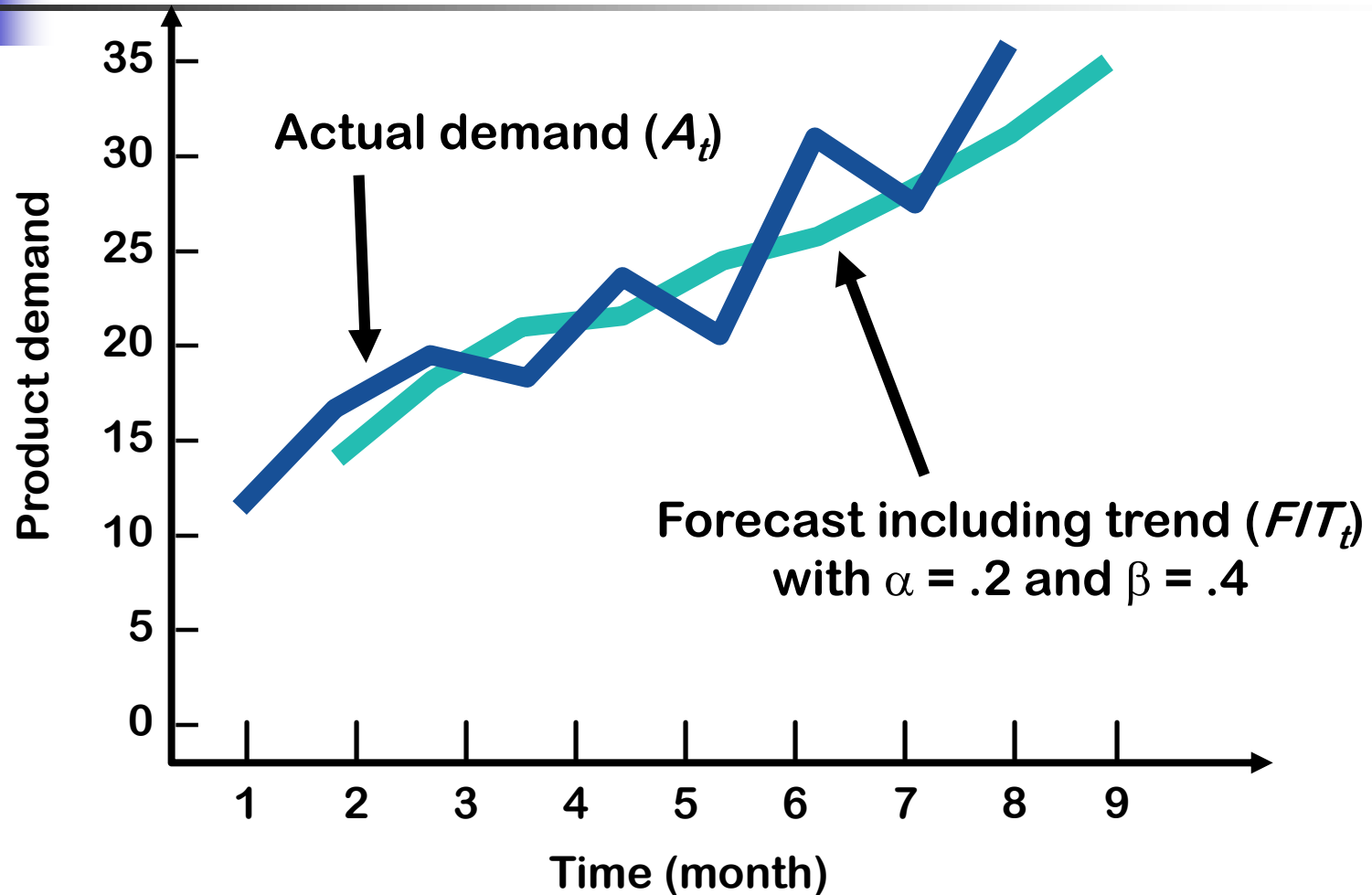
$$\begin{aligned}
 FIT_2 &= F_2 + T_2 \\
 FIT_2 &= 12.8 + 1.92 \\
 &= 14.72 \text{ units}
 \end{aligned}$$

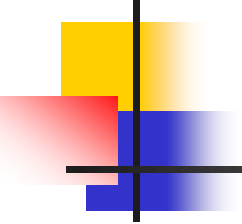


Exponential Smoothing with Trend Adjustment

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89
7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10		32.48	2.68	35.16

Exponential Smoothing with Trend Adjustment





Thank you

